Inductive MHD Generator*

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(Z. Naturforschg. 18 a, 885—889 [1963]; eingegangen am 17. Mai 1963)

The electrodynamic response of a particular inductive magnetohydrodynamic generator is calculated. Expressions for the output voltage and output power are derived. The results of experiments conducted with the generator mounted on an electromagnetic shock-tube agree with the theoretical predictions. The measured output power is of the order of 10 watts lasting for a few microseconds.

The problems concerned with sheath potentials, the deposition of seeding materials, and the erosion of electrodes and electrical insulation are not encountered with electrodeless MHD generators. In one such generator the load is coupled inductively to the magnetic induction field generated by currents flowing in the plasma. One particular configuration of inductive MHD generator is considered in the present work. This configuration, shown in Fig. 1,

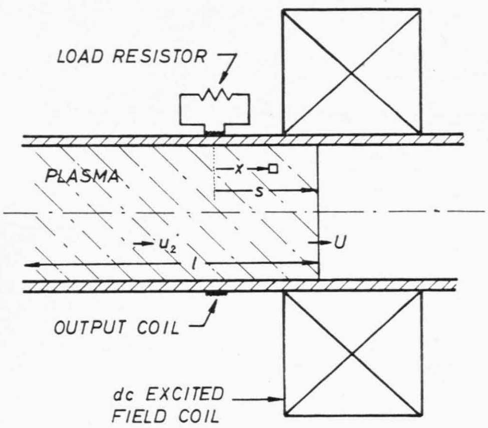

Fig. 1. Inductive MHD generator.

has been employed by Lin, Resler and Kantrowitz [1] to measure the electrical conductivity of the plasma generated in a conventional diaphragm-type shock tube, and has been considered by Woodson and Lewis [2] as a possible MHD generator. In the latter work the effect of a high magnetic Reynold's number on the transport of magnetic flux in the plasma was investigated. In the present work the primary object is the determination of the power generating characteristics of this generator. The time response of the output voltage and power is calculated and compared with experimental results.

Theory

The analysis given here closely follows that of Lin et al. [1] and the same notation is used. In their work the load resistance was approximately equal to the critical damping resistance of the output coil. Such a high value of load resistance resulted in a very low output power, but an output voltage that was essentially independant of the load resistance. In the analysis to follow, the effect of the load resistance R and output coil inductance L_1 on the output voltage V_1 and output power P_1 will be determined.

Following Lin et al. [1] it is assumed that the flow velocity is uniform, that there is no transverse motion of gas and that there is no change in the current through the field coil during the interaction. The electric field in the plasma is neglected and also it assumed that the magnetic field acting on the plasma is solely that due to the field coils.

Kirchhoff's Law for the load circuit is

$$I_1 R_1 + L_1 \frac{dI_1}{dt} + A \frac{d\Phi}{dt} = 0 \tag{1}$$

where I_1 is the load current, A is the cross-sectional area of the load coil and Φ is the total induced magnetic flux passing through the load coil. This equation can be written in a form like Lin et al's. [1].

* This work was performed at the University of British Columbia, Vancouver, Canada.

[1] S. C. Lin, E. L. Resler and A. Kantrowitz, J. Appl. Phys. 26, 95 [1955].

[2] H. H. Woodson and A. T. Lewis, Engineering Aspects of Magnetohydrodynamics, Columbia University Press 1962, p. 277.

$$I_l R_l + L_l \frac{dI_l}{dt} = -A\, U\, u_2\, I \left\{ \sigma(o)\, g(s) - \sigma(l)\, g(s-l) + \int_{s-l}^{s} \sigma'(s-x)\, g(s-x)\, dx \right\}. \tag{2}$$

The function g can be eliminated by following the calibration procedure of LIN et al. [1]. In this method a slug of metal of known conductivity σ_c is passed through the coils at velocity u_c. If the field coil current is I_c while the slug is passing through the coils, then the voltage induced in the load coil is $V_c(s)$. Equation (2) can be rewritten as

$$I_l R_l + L_l \frac{dI_l}{dt} = \frac{-U\, u_2\, I}{u_c^2\, I_c\, \sigma_c} \left\{ \sigma(o)\, V_c(s) - \sigma(l)\, V_c(s-l) + \int_{s-l}^{s} \sigma'(s-x)\, V_c(x)\, dx \right\} \tag{3}$$

wich is equivalent to

$$I_l = \frac{-U\, u_2\, I}{R_l\, I_c\, \sigma_c\, u_c^2} \exp\left\{-\frac{R_l}{L_l} t\right\} \int_{-\infty}^{t} \left\{\exp \frac{R_l}{L_l} t\right\} \left\{ \sigma(o)\, V_c(s) - \sigma(l)\, V_c(s-l) + \int_{s-l}^{s} \sigma'(s-x)\, V_c(x)\, dx \right\} dt. \tag{4}$$

The solution to equation (4) for a step increase in conductivity at the shock front and a coil response function $V_c(s)$ of the form of a GAUSSian distribution is of particular interest. The validity of such a solution could readily be checked experimentally since the plasma generated in a shock tube approximately possesses such a step increase in conductivity and the experimentally-observed coil response function is quite closely that of a GAUSSian distribution function. For these assumptions,

$$V_c(s) = V_{cp} \exp\left\{-\left(\frac{s-s_b}{b}\right)^2\right\}, \qquad \sigma(s=0) = \sigma_p, \qquad \sigma'(s-x) = 0. \tag{5}$$

The calculation is being made only for the shock front, not the trailing portion of the plasma.

The solution to equation (4) under these assumptions is

$$I_l(t) = \frac{-U\, I\, V_{cp}\, u_2\, \sigma_p}{u_c^2\, \sigma_c\, I_c\, L_c} \exp\left\{-\frac{R_l}{L_l} t\right\} \int_{-\infty}^{t} \exp\left\{\frac{R_l}{L_l} t - \frac{U^2}{b^2} t^2\right\} dt \tag{6}$$

where the substitution $s = U\, t + s_b$ has been employed. The origin of time is thus chosen as the time when the plasma front is half way through te load coil. Performing the integration yields

$$I_l(t) = \frac{-\sqrt{\pi}\, b\, u_2\, \sigma_p\, I\, V_{cp}}{2\, \sigma_c\, u_c^2\, I_c\, L_l} \left\{\exp\left\{\frac{R_l^2\, b^2}{4\, L_l^2\, U^2} - \frac{R_l}{L_l} t\right\}\right\} \left\{1 + \operatorname{erf} \frac{U}{b}\left[t - \frac{R_l\, b^2}{2\, U^2\, L_l}\right]\right\}. \tag{7}$$

The voltage appearing across R_l is given by

$$V_l = I_l\, R_l. \tag{8}$$

It can be shown that a criterion for the neglect of the effect of R_l on V_l is that $R_l\, b^2/2\, L_l\, U^2 \gg |t|$. When this criterion is satisfied, equation (7) reduces to a GAUSSian distribution, the special case considered by LIN et al. [1].

The power in the load is

$$P_l(t) = [I_l(t)]^2\, R_l \tag{9}$$

or

$$P_l(t) = \left\{\frac{u_2\, \sigma_p\, I\, V_{cp}}{u_c^2\, \sigma_c\, I_c}\right\}^2 \frac{\pi\, R_l\, b^2}{4\, L_l^2} \left\{\exp\left\{\frac{R_l^2\, b^2}{2\, L_l^2\, U^2} - \frac{2\, R_l\, t}{L_l}\right\}\right\} \left\{1 + \operatorname{erf} \frac{U}{b}\left[t - \frac{R_l\, b^2}{2\, L_l\, U^2}\right]\right\}^2. \tag{10}$$

To obtain the conditions under which maximum energy dissipation in the load would occur, it would be necessary to maximize

$$\int_{-\infty}^{+\infty} P_l(t)\, dt. \tag{11}$$

This equation has not been solved. An estimate of the peak value of $P_l(t)$ can, however, be obtained from equation (10) assuming $t = 0$. The resulting equation can be solved graphically for the value of the load resistance R_l that maximizes $P_l(t=0)$. The value thus obtained is $R_l^* = 1.36\, L_l\, U/b$. Then

$$P_l(t=0,\, R_l = R_l^*) = 0.415\, [V_p^*]^2/R_l^* \tag{12}$$

where $V_p^* = U\, u_2\, \sigma_p\, I\, V_{cp}/u_c^2\, \sigma_c\, I_c$. A graph of

$$P_l(t,\, R_l = R_l^*), \quad U/b = 10^6\ \mathrm{sec}^{-1},$$

$$K_1 = [u_2\, \sigma_p\, I\, V_{cp}/u_c^2\, \sigma_c\, I_c]^2\, \pi\, b\, U/2\, L_l$$

is given in Fig. 2.

A numerical value for $P_1(0)$ that is representative of the value expected on a shock tube is $P_1(0) \approx 7.5$ W. The assumed conditions are $U = u_2 = 10^4$ m/sec, $b = 0.01$ m, $\sigma_p = 10^4$ mhos/m, $I_c u_c^2 / V_{cp} = 1.8 \times 10^5$ amp m²/sec² volt, $R_1 b / L_1 U = 1.36$, $L_1 = 35$ nH, $I = 10$ amp, $\sigma_c = .58 \times 10^8$ mhos/m, $R_1 = .0475\ \Omega$. An output power of 7.5 W lasting for a time of the order of a microsecond is insufficient to slow the plasma. The electrodynamic analysis presented above is thus sufficient to describe the operation of this type of MHD generator. A magnetohydrodynamic analysis is not warranted unless a larger output power could be obtained. Multiple output coils and loads would be one method of increasing the output power.

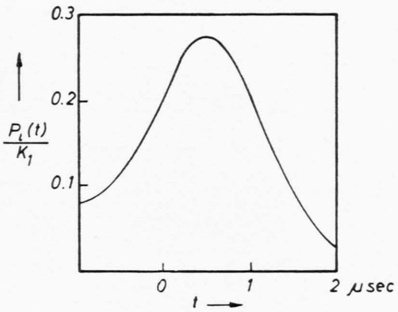

Fig. 2. Power dissipated in load resistance, theoretical.

Experimental Results

The validity of the above theory was checked experimentally. The apparatus used is shown in Fig. 3. The field coil had an inner diameter of 3.5 cm, an outer diameter of 11.7 cm and a length of 2.5 cm. This coil was wound with 296 turns of wire 1.6 mm in diameter. The single layer output coil was wound with 6 turns of 0.51 mm diameter wire on a coil form 3.12 cm diameter and 3.9 mm in length. The coils were calibrated by a method similar to that used by LIN et al. [1]. The results of calibration were: $b = .02$ m, $I_c u_c^2 / V_{cp} = 7.50 \times 10^4$ amp m²/sec² volt, $\sigma_c = 0.581 \times 10^8$ mhos/m.

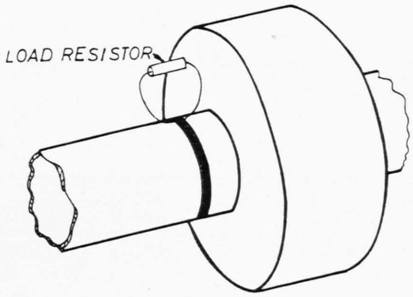

Fig. 3. Experimental apparatus.

As is shown in Fig. 3, the load, a one turn coil, was oriented so that magnetic flux generated by azimuthal current in the plasma did not link with the load. The current through the load was thus generated only by the rate of change of magnetic flux linking with the output coil. The resistance of the output coil (0.05 Ω) was negligible compared with the load resistance. The inductance of the 6 turn output coil was calculated as 1.91 μH, much larger than the calculated value of the inductance of the load, 70 nH. The output coil was thus assumed to have only an inductive impedance and the load only a resistive impedance. The equivalent circuit was therefore as shown in Fig. 4. The value of C, as determined from the oscillatory frequency of the unloaded coil, 33 Mc/s, was 12 pF. The load resistance for critical damping was thus 800 Ω.

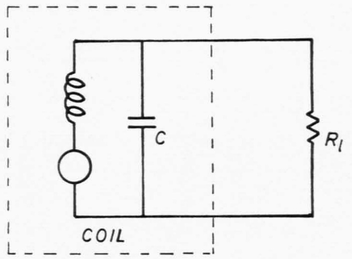

Fig. 4. Equivalent circuit for output coil of experimental apparatus.

The curves shown in Fig. 5 for $R_1 = 3.25\ \Omega$ and $R_1 = 0.50\ \Omega$ were displaced spuriously late in time. The signals observed with an RCA type 931-A photomultiplier, viewing transverse to the shock tube and at a

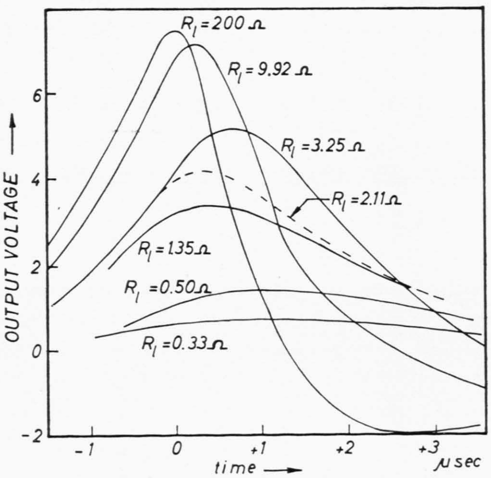

Fig. 5. Voltage measured across load resistance.

[3] G. D. CORMACK, Ph. D. Thesis, University of British Columbia 1962.

distance of 38 cm from the base of the driver, were not of the usual flat-top waveshape for these two shots. The deviation from the usually-observed flat-top waveshape and the displacement in time of the coil were considered to be caused by the shot-to-shot irreproducibility of the electromagnetic driving mechanism that produced the plasma. This irreproducibility characteristic has been investigated by CORMACK [3].

The power dissipation curves shown in Fig. 6 were obtained from the data shown in Fig. 5. The theoretical value of load resistance that optimized $P_l(t=0)$ was $1.36\, L_l\, U/b = 2.7\, \Omega$, and the maximum value of $P_l(t=0)$

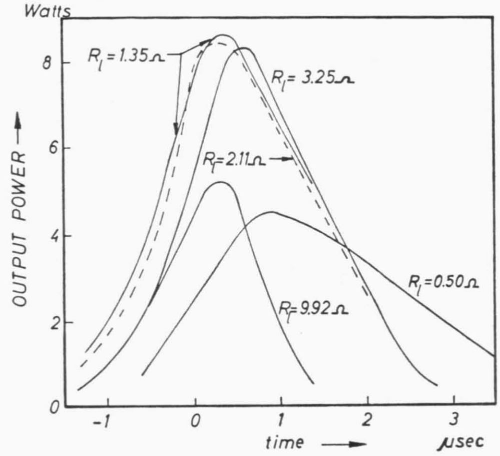

Fig. 6. Power dissipation in load resistance.

was from equation (12), 8.6 W. The conductivity of the plasma was assumed to be 10^4 mhos/m (see Appendix). From Fig. 6 the experimentally-determined maximum value of $P_l(t=0)$ was 7.8 W, obtained when $R_l = 2.2 \pm 1\, \Omega$. The predicted and the observed values of $P_l(t=0, R_l = R_l^*)$ and of R_l^* were thus in agreement. Some error was introduced by the assumption that the plasma possessed a step function of conductivity. An analysis that is more applicable for the observed waveforms could be made for a conductivity function that is a step increase followed by an exponential decay. For this case the resulting integral for $I_l(t)$ can be solved in closed form only if R_l is large. This analysis is presented in the Appendix.

Discussion

The observed response of the generator is in accord with the theory. The fairly close agreement indicates that the assumptions made in the analysis were justified. It has been found that this generator is not a practical power generator. An output power of 10 W is very low in comparison with that which has been obtained from electrode-type MHD generators, for example by PAIN and SMY [4].

Acknowledgments

The author wishes to thank Dr. A. J. BARNARD for giving guidance in this research. Financial assistance was provided by The National Research Council of Canada and the Atomic Energy Control Board.

Appendix

The curves shown in Fig. 5 indicate that the conductivity of the plasma being studied is not described by equation (5). The overshoot on the signals indicates that the conductivity of the plasma is quite closely described by

$$\sigma(\xi) = \sigma^* e^{-\xi/\beta}, \quad \xi > 0, \qquad \sigma(\xi) = 0 \quad , \quad \xi < 0. \tag{13}$$

Substituting equations (8) and (13) into equation (3), letting $L_l = 0$ and solving yields

$$V_l(t) = \frac{U\, u_2\, I\, \sigma^*\, V_{cp}}{u_c^2\, I_c\, \sigma_c}\, \psi(t) \tag{14}$$

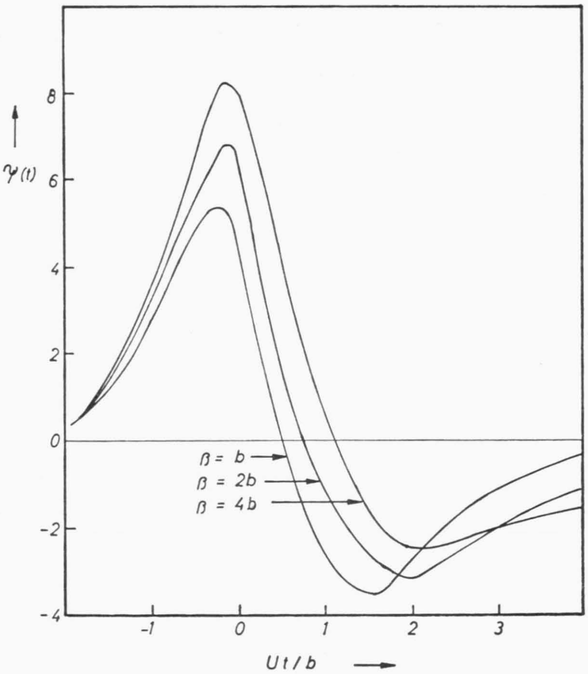

Fig. 7. Calculated waveform of output voltage when the load resistance is large and the conductivity function that of equation (13).

where

$$\psi(t) = \exp\left\{\left[\frac{U\,t}{b}\right]^2\right\} - \frac{\sqrt{\pi}\, b}{2\beta}\left\{\exp\left\{\left[\frac{b}{2\beta}\right]^2 - \frac{U\,t}{b}\right\}\right\}\left\{1 + \operatorname{erf}\left[\frac{U\,t}{b} - \frac{b}{2\beta}\right]\right\}. \tag{15}$$

[4] H. J. PAIN and P. R. SMY, J. Fluid Mech. 10, 51 [1961].

This equation is, of course, only valid if $R_l\, b^2/2\, L_l\, U^2 \gg |t|$. The expression for $V_l(t)$ for any value of R_l is found from equation (4) to be

$$V_l(t) = \frac{-U\, u_2\, I\, \sigma^*\, V_{cp}\, R_l}{u_c^2\, I_c\, \sigma_c\, L_l} \exp\left\{-\frac{R_l}{L_l} t\right\} \int_{-\infty}^{t} \left\{\exp + \frac{R_l}{L_l} t\right\} \left\{ \exp\left\{-\left[\frac{U t}{b}\right]^2\right\} - \frac{1}{\beta} \int_{-\infty}^{U t} \exp\left\{-\left[\frac{x}{b}\right]^2 - \left[\frac{s-x}{\beta}\right]\right\} dx \right\} dt\,. \quad (16)$$

Equation (16) cannot be solved by analytical methods.

The form of the function $\psi(t)$ in equation (15) for various values of β is shown in Fig. 7. The observed waveform, the $R_l = 200\ \Omega$ curve in Fig. 5, is quite closely of the same shape as the $\beta = 4\, b$ curve in Fig. 7. The maximum conductivity of the plasma is given by

$$\sigma^* = \left\{\frac{u_c^2\, I_c\, \sigma_c}{V_{cp}}\right\} \left\{\frac{V_p}{U\, u_2\, I}\right\} \left\{\frac{1}{\psi_1}\right\} \quad (17)$$

where ψ_1 is the maximum value of $\psi(t)$ and V_p is the peak value of voltage observed. Thus for $U = u_1 = 2 \times 10^4$ m/sec, $u_c^2\, I_c/V_{cp} = 7.5 \times 10^4$ amps m²/sec² volt, $\sigma_c = 0.581 \times 10^8$ mhos/m; $V_p = 7.5$ V, $I = 10$ A, $\psi_1 = 0.82$, then $\sigma^* = 10^4$ mhos/m.

Diffusion eines Plasmas in Abhängigkeit von Magnetfeld und Druck und von der Längs-Ausdehnung in Magnetfeldrichtung*

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The radial density distribution for a plasma in a uniform magnetic field was studied in dependence of pressure and distance of the conducting end plates. It was possible to confirm experimentally the dependence of the radial distribution of the finite length in direction of the field lines. The influence of the magnetic field, of the pressure, and of the length of the plasma column on the radial density profile is, in different gases, qualitatively in accordance with the "short-circuiting" theory of A. Simon.

Die Diffusion von Ladungsträgern senkrecht zum Magnetfeld ist ein fundamentales Problem der Plasmaphysik, für das bis jetzt kaum eine wirklich zufriedenstellende Übereinstimmung zwischen Theorie und Experiment erreicht werden konnte. Der Grund ist darin zu suchen, daß die „normale" Stoß-Diffusion, bei der die Ladungsträger im Mittel einen Gyro-Radius pro Stoß versetzt werden, offensichtlich der „langsamste" aller denkbaren Transport-Mechanismen ist. Statische und oszillierende elektrische Felder oder ganz allgemein Instabilitäten können eine um Größenordnungen höhere „anomale" Diffusion verursachen. Experimentell ist es sehr schwierig, ein ruhiges Plasma mit einer Maxwell-Verteilung der Ionen und Elektronen zu erzeugen, das Vergleichsmöglichkeiten mit der Theorie bietet. Wenn aber unter sorgfältig kontrollierten Bedingungen die „normale" Diffusion in einem Experiment einmal bestätigt worden ist, kann durch sinnvolle Änderung der Parameter das Einsetzen von Instabilitäten und ihre Auswirkung auf den Transport von Ladungsträgern senkrecht zum Magnetfeld systematisch untersucht werden.

Beim Vergleich mit der Theorie wird angenommen, daß das Plasma in der Zylinderachse eines homogenen Magnetfeldes erzeugt wird. Das sekundäre Plasma, das durch Diffusion aus der Achse entsteht, sollte die Forderungen nach einem ruhigen Plasma mit Maxwell-Verteilung relativ gut erfüllen, da keine äußeren Felder die Diffusion beeinflussen und die Ladungsträger – insbesondere die Elektronen – eine große Anzahl von Stößen mit den neutralen Gas-Molekülen erleiden.

Unter dem Einfluß von Dichtegradienten und elektrischen Feldern strömen die Ladungsträger parallel und senkrecht zum Magnetfeld zu den Wänden der Vakuumkammer. Für den zwei-dimensionalen Fall, bei dem das Magnetfeld in z-Richtung angelegt ist, während die r-Richtung senkrecht zum Magnetfeld liegt, ist dieser Teilchenfluß durch folgende Glei-

* Diese Arbeit wurde im Rahmen des Vertrages zwischen dem Institut für Plasmaphysik GmbH und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

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